

THE CALGARY MATHEMATICAL ASSOCIATION  
**30<sup>TH</sup> JUNIOR HIGH SCHOOL MATHEMATICS CONTEST**  
 April 26, 2006

NAME: \_\_\_\_\_  
 PLEASE PRINT (First name Last name)

GENDER:  M  F

SCHOOL: \_\_\_\_\_

GRADE: \_\_\_\_\_  
 (7,8,9)

- You have 90 minutes for the examination. The test has two parts: PART A — short answer; and PART B — long answer. The exam has 9 pages including this one.
- Each correct answer to PART A will score 5 points. You must put the answer in the space provided. No part marks are given.
- Each problem in PART B carries 9 points. You should show all your work. Some credit for each problem is based on the clarity and completeness of your answer. You should make it clear why the answer is correct.

PART A has a total possible score of 45 points.

PART B has a total possible score of 54 points.

- You are permitted the use of rough paper. Geometry instruments are not necessary. References including mathematical tables and formula sheets are **not** permitted. Simple calculators without programming or graphic capabilities **are** allowed. Diagrams are not drawn to scale. They are intended as visual hints only.
- When the teacher tells you to start work you should read all the problems and select those you have the best chance to do first. You should answer as many problems as possible, but you may not have time to answer all the problems.

**BE SURE TO MARK YOUR NAME AND SCHOOL AT THE TOP OF THIS PAGE.**  
**THE EXAM HAS 9 PAGES INCLUDING THIS COVER PAGE.**

**Please return the entire exam to your supervising teacher at the end of 90 minutes.**

| MARKERS' USE ONLY |    |    |    |    |    |    |                    |
|-------------------|----|----|----|----|----|----|--------------------|
| PART A _____×5    | B1 | B2 | B3 | B4 | B5 | B6 | TOTAL<br>(max: 99) |
|                   |    |    |    |    |    |    |                    |

**PART A: SHORT ANSWER QUESTIONS**

**A1** A prime number plus a perfect square equals 99. What is the prime number?

**A2** The price of a TV (before tax) is a whole number of dollars and is the same in Alberta and in BC. That tax is 7% in Alberta and 15% in BC. After the tax is applied the TV costs \$10 more in BC than in Alberta. What is the before-tax price (in dollars) of a TV?

**A3** A quadrilateral has three sides of lengths 5.5, 6.5 and 7.5 metres. The length of the fourth side in metres is a positive integer. How many possible lengths (in metres) can the fourth side have?

**A4** Reim and Bindu are in a line with other students, waiting to see a movie. There are at most 30 students in the line.

*Reim says:* "There are three times as many students after me in this line than before me."

*Bindu says:* "There are four times as many students after me in this line than before me."

How many students are in the line?

**A5** Anna, Bob and Carol ran a 1000m race and each ran at a (different) constant speed throughout the entire race. When Anna finished, Bob and Carol were at the 800m and 600m mark respectively. When Bob finished, how far (in metres) was Bob ahead of Carol?

- A6. Thirty-one students who write a contest get all the integer grades from 70 through 100, with different students getting different grades. When one particular score is removed, the average of the 30 remaining scores is the same as the average of all 31 scores. What score has been removed?

- A7 Robert was reading a book and was counting the number of 1s that appeared in the page numbers. He counted that there were 24 ones. If the book starts on page 1, how many pages does the book contain?

- A8 All of the possible arrangements of the letters *MATH* are used to form four letter codes. These codes are put in a list in alphabetical order. (So the first code in the list is *AHMT*.) What is the 7th code in this list?

- A9 The number  $N = 111\dots 1$  consists of 2006 ones. It is exactly divisible by 11. How many zeroes are there in the quotient  $\frac{N}{11}$ ?

## PART B: LONG ANSWER QUESTIONS

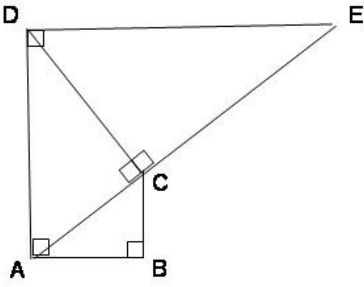
**B1** Silvia needs to buy two shirts. Two stores, Shirt Check and Supershirts, sell the shirts she is searching for. Shirt Check's regular price is \$5 more than Supershirts'. However, Shirt Check has a special where if you buy one shirt at the regular price, you get the second shirt at 40% off the regular price. Supershirts is selling every shirt at 10% off the regular price. It turns out that the two shirts would cost Silvia exactly the same at Shirt Check as at Supershirts. What is the regular price of a shirt at Shirt Check?

**B2** Alex swims  $2\frac{1}{2}$  times as fast as Boris. They start together at one end of the pool and swim back and forth from one end to the other. The swimming pool is 25m long. Boris swims 30 lengths of the pool (750 m) and then stops. How many times has Alex passed Boris, either going in the same direction or in the opposite direction? (If Alex and Boris arrive at one end of the pool at the same time, it counts as a pass. But do not count the beginning when they start together.)

**B3** In the figure,  $AB = 4$ ,  $BC = 3$ , and

$$\angle ABC = 90^\circ = \angle ACD = \angle DCE = \angle ADE = \angle DAB.$$

Find the length of  $AE$ .



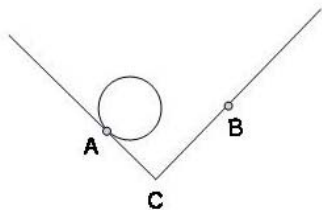
**B4** Let's say that an integer  $N > 1$  is *friendly* if every time  $N$  is written as the sum of *two* positive integers  $A$  and  $B$ , some digit of  $A$  or  $B$  is also a digit of  $N$ : that is,  $N$  **cannot** be written as the sum of two positive integers which do not use any of the digits of  $N$ . For example, 120 is **not** friendly, because you can write 120 as a sum of two positive integers without using the digits 1, 2 or 0: for instance you could write 120 as  $76 + 44$ .

(a) Show that 2006 is not friendly.

(b) Find an integer  $N > 2006$  that is friendly. Make sure to say why you know your number is friendly.

(c) Let's say that an integer  $N > 1$  is *really friendly* if  $N$  cannot be written as the sum of two, or three, or four, or **any** number of positive integers without using at least one of the digits of  $N$ . Find a really friendly integer bigger than 1 but smaller than 100000. Make sure to say why you know your number is really friendly.

- B5** A wheel with radius 1 metre is rolled down one side of a right-angled trough (as in the diagram) and up the other side, without slipping. It runs over a blob of paint at point  $A$  on the first side. After that, every time that point on the wheel hits the trough it makes a paint mark on the trough. The first time this happens is at point  $B$  on the other side. The paint spot  $A$  on the first side is 2 metres from the corner  $C$  of the trough. How far from the corner is the paint mark  $B$ ?





**B6** Find all positive integers  $a$  and  $b$  so that

$$\frac{a}{b} - \frac{a+1}{b+1} = \frac{a+2}{b+2}.$$

Make sure to prove that you have found all solutions.